Math 2550 - Homework # 9

Matrices of Linear Transformations

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1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ where $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ -2x + 4y \end{pmatrix}$.

   Let $\beta = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right]$

   One can show that β is a basis for $\mathbb{R}^2$. You don't have to check it.

   (a) Show that T is a linear transformation.

   (b) Find $[T]_\beta$.

   (c) Verify that $[T]_\beta[\vec{v}]_\beta = [T(\vec{v})]_\beta$ for $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

   (d) Verify that $[T]_\beta[\vec{v}]_\beta = [T(\vec{v})]_\beta$ for $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

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2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ where $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x + y \end{pmatrix}$.

   Let $\beta = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right]$

   One can show that β is a basis for $\mathbb{R}^2$. You don't have to check it.

   (a) Show that T is a linear transformation.

   (b) Find $[T]_\beta$.

   (c) Verify that $[T]_\beta[\vec{v}]_\beta = [T(\vec{v})]_\beta$ for $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

   (d) Verify that $[T]_\beta[\vec{v}]_\beta = [T(\vec{v})]_\beta$ for $\vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ where

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4x + z \\ 2x + 3y + 2z \\ x + 4z \end{pmatrix}$$

In the previous HW you showed $\lambda = 3$ is an eigenvalue of T with eigenvectors $\vec{a} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and that $\lambda = 5$ is an eigenvalue of T with eigenvector $\vec{c} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$. Let $\beta = [\vec{a}, \vec{b}, \vec{c}]$. One can show that the vectors in β are linearly independent and hence are a basis for $\mathbb{R}^3$.

(a) Find $[T]_\beta$

(b) Verify that $[T(\vec{v})]_\beta = [T]_\beta [\vec{v}]_\beta$ using $\vec{v} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$

[Hint: $\vec{v} = 1 \cdot \vec{a} + 1 \cdot \vec{b} + 1 \cdot \vec{c}$]